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A method for evaluation of heat and mass transport properties of moist porous media

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Abstract—A method is proposed for evaluation of the mass diffusivity $D_m(w,t)$, thermo-mass diffusivity $D_t(w,t)$, non-linear thermal conductivity $\xi(w,t)$, and mass–thermal conductance $\alpha(w,t)$ of moist porous media at different temperatures and moisture contents based on the measured mass diffusivity $D_m(w,t^*)$ and thermo-mass diffusivity $D_t(w,t^*)$ at a specified reference temperature t^*. It is a valuable and convenient approach for engineering practice.

1. INTRODUCTION

THE HEAT and mass transfer in moist porous media often occurs in nature and in engineering. The case without considering the total pressure gradient is a special one, of which the transport phenomena in soils or in building materials are typical examples [1, 2]. The heat and mass transport properties of moist porous media are basically important in solving such problems [3]. However, there are very little data and still few convenient means available for the measurement of these properties.

A synthetical theory, based on a previous paper [4], was developed further for heat and mass transfer phenomena in unsaturated, moist porous media without sweeping flow of fluids [5, 6], which relates the transport properties to the characteristic functions of the basic transfer mechanisms, and presents the heat and mass transfer equations as follows:

$$\rho C_p^* \frac{\partial t}{\partial \tau} = \nabla \cdot (\lambda^* \nabla t) + \xi (\nabla t)^2 + \alpha \nabla w \cdot \nabla t \qquad (1)$$

$$\frac{\partial w}{\partial \tau} = \nabla \cdot (D_m \nabla w) + \nabla \cdot (D_t \nabla t) \qquad (2)$$

where t and w denote local temperature and moisture content, respectively, ρ the local density of the moist porous body, C_p^* the nominal specific heat, λ^* the nominal thermal conductivity, ξ the non-linear thermal conductivity, α the mass–thermal conductance, and D_m and D_t are mass diffusivity and thermo-mass diffusivity, respectively. It would be very difficult to experimentally determine all the properties included in equations (1) and (2). With the assumption of constant properties and small temperature and moisture gradients, it was shown [6] that equation (1) can be simplified to

$$\frac{\partial t}{\partial \tau} = a_e \nabla^2 t \qquad (3)$$

where $a_e = \lambda_e/(\rho C_p^*)$ is the 'effective' thermal diffusivity, and λ_e the 'effective' thermal conductivity which reflects the overall effect of nominal heat conduction indicated by nominal thermal conductivity, vapour diffusion, and capillary flow of liquid on heat transfer, just the same as that derived in ref. [4].

Recently, we had reported a method for measuring simultaneously the mass diffusivity D_m, and the thermo-mass diffusivity D_t with a one-dimensional constant-property system subject to the Robin boundary condition [7], of which the accuracy depends mainly on the stability of surrounding conditions. A method for measuring the mass diffusivity for moisture migration in porous media from the isothermal water absorption with consideration of variable properties was also proposed [8]. Even for D_m and D_t, it is still not an easy task to determine them in a larger temperature range. Here, we propose a method to evaluate the heat and mass transport properties of moist porous media at different temperatures in the range $0 < t < 100°C$, based on the measured $D_m(w,t^*)$ and $D_t(w,t^*)$ data at a specified reference temperature t^*. It would be obviously significant for engineering applications.

2. MAIN IDEA

The relations between the transport properties and the characteristic functions of basic transfer mechanisms were derived in ref. [6] and summarized in ref. [5]. For convenience, they are reorganized and rewritten as

$$D_m(w,t) = -\frac{k(w)}{\mu(t)} \frac{\partial p_c}{\partial w}(w,t) \qquad (4)$$

$$D_t(w,t) = B(w)D(t)\frac{d\omega}{dt} - \frac{k(w)}{\mu(t)} \frac{\partial P_c}{\partial t}(w,t) \qquad (5)$$

$$\xi(w,t) = \rho_0 \left[(C_{pv} - C_{pa}) B(w) D(t) \frac{d\omega}{dt} - C_{pl} \frac{k(w)}{\mu(t)} \frac{\partial p_c}{\partial t}(w,t) \right] \qquad (6)$$

NOMENCLATURE

a_e	effective thermal diffusivity [$m^2 s^{-1}$]
B	influence function
C_p	specific heat [$J kg^{-1} K^{-1}$]
C_p^*	nominal specific heat [$J kg^{-1} K^{-1}$]
D	diffusion coefficient for vapour–air diffusion [$m^2 s^{-1}$]
D_m	mass diffusivity [$m^2 s^{-1}$]
D_t	thermo-mass diffusivity [$m^2 s^{-1} K^{-1}$]
k	specific permeability [m^2]
p_c	capillary pressure [$N m^{-2}$]
t	temperature [°C]
T	temperature [K]
w	moisture content [kg (moisture)/kg (solid matrix)].

Greek symbols

α	mass–thermal conductance [$W m^{-1} K^{-1}$]
λ_e	effective thermal conductivity [$W m^{-1} K^{-1}$]
λ^*	nominal thermal conductivity [$W m^{-1} K^{-1}$]
μ	dynamic viscosity of wetting fluid [$N s m^{-2}$]
ξ	non-linear thermal conductivity [$W m^{-1} K^{-2}$]
ρ	density [$kg m^{-3}$]
σ	surface tension [$N m^{-1}$]
τ	time [s]
ω	mass content of water vapour in saturated humid air [kg (water vapour)/kg (humid air)].

Subscripts

a	air
l	liquid water
s	saturated state
v	water vapour
0	solid matrix, or at 0°C.

$$\alpha(w,t) = -\rho_0 C_{pl}\frac{k(w)}{\mu(t)}\frac{\partial p_c}{\partial w}(w,t) \tag{7}$$

where p_c is the capillary pressure of moist porous media, μ the dynamic viscosity of the wetting fluid, ω the mass content of water vapour in saturated humid air, D the diffusion coefficient for vapour–air diffusion, C_p the specific heat of fluids at constant pressure, the subscripts a, v and l of which denote air, water vapour and liquid water, respectively, ρ_0 the nominal density of the solid matrix, k the specific permeability which is a function of moisture saturation only [9], and B reflects the influence of moisture content on vapour diffusion in porous media.

For moist porous bodies, the capillary pressure p_c is varied with temperature and moisture content according to [10]

$$p_c(w,t) = p_0\frac{\sigma(t)}{\sigma_0}\left(\frac{w}{w_s}\right)^{-n} \tag{8}$$

where w_s denotes the saturated moisture content, σ the surface tension, and σ_0 the surface tension when the temperature approaches 0°C. As given by Zemansky and Dittman [5]

$$\sigma(t) = \sigma_0\left(1-\frac{t}{t_c}\right)^m. \tag{9}$$

For the water–air interface, $t_c = 374$°C, $m = 1.2$. Substituting equation (9) into equation (8), we have the derivatives

$$\frac{\partial p_c}{\partial w} = -\frac{np_0}{w_s}\left(1-\frac{t}{t_c}\right)^m\left(\frac{w}{w_s}\right)^{-n-1} \tag{10}$$

$$\frac{\partial p_c}{\partial t} = -\frac{mp_0}{t_c}\left(1-\frac{t}{t_c}\right)^{m-1}\left(\frac{w}{w_s}\right)^{-n} \tag{11}$$

where p_0 is the saturated capillary pressure when the temperature approaches 0°C, the power n may differ for different porous media. Both p_0 and n can be determined in hydrodynamics and infiltration mechanics with effective methods.

Substituting equations (10) and (11) into equations (4)–(7), we obtain the following practical relations:

$$D_m(w,t) = \frac{np_0}{w_s}\frac{k(w)}{\mu(t)}\left(1-\frac{t}{t_c}\right)^m\left(\frac{w}{w_s}\right)^{-n-1} \tag{12}$$

$$D_t(w,t) = f_1(w,t)+f_2(w,t) \tag{13}$$

$$\xi(w,t) = \rho_0[(C_{pv}-C_{pa})f_1(w,t)+C_{pl}f_2(w,t)] \tag{14}$$

$$\alpha(w,t) = \rho_0 C_{pl} D_m(w,t) \tag{15}$$

with $f_1(w,t)$ and $f_2(w,t)$ defined as

$$f_1(w,t) = B(w)D(t)\frac{d\omega}{dt} \tag{16}$$

$$f_2(w,t) = \frac{m}{n(t_c-t)}wD_m(w,t). \tag{17}$$

Both $k(w)$ and $B(w)$ can be determined from the following relations:

$$k(w) = \frac{\mu(t^*)w_s}{np_0}\left(1-\frac{t^*}{t_c}\right)^{-m}\left(\frac{w}{w_s}\right)^{n+1}D_m(w,t^*) \tag{18}$$

$$B(w) = \frac{D_t(w,t^*)-f_2(w,t^*)}{D(t^*)\dfrac{d\omega(t^*)}{dt}}. \tag{19}$$

3. EVALUATING APPROACH

For the porous body of pore diameter larger than 10^{-5} cm, the gas phase diffusion can be approximately described by Fick's law of diffusion [2], so, the diffusion coefficient can be chosen as [12]

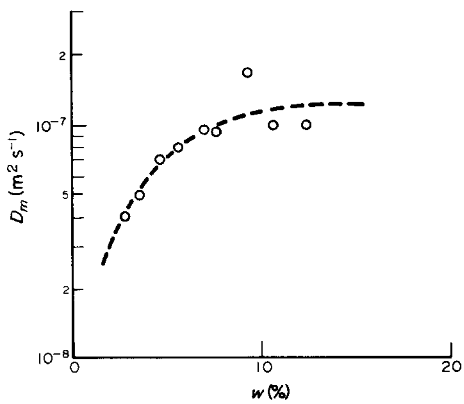

FIG. 1. The measured mass diffusivity of the sand at 20°C.

$$D(t) = 2.23 \times 10^{-5}\left(\frac{T}{256}\right)^{1.81} \ [\mathrm{m^2\,s^{-1}}] \quad (20)$$

where T is the absolute temperature.

The mass content of water vapour in saturated humid air under 760 mmHg with the variation of temperature, $\omega(t)$, was derived as [10]

$$\frac{d\omega}{dt} = \frac{1.8041 \times 10^6 \exp(g)}{[760 - 0.378\exp(g)]^2(t + 227.02)^2} \ [\mathrm{K^{-1}}] \quad (21)$$

with the parameter g as

$$g(t) = 18.304 - \frac{3816.4}{t + 227.02}. \quad (22)$$

The dynamic viscosity of liquid water, $\mu(t)$, can be easily predicted from the well-known empirical expressions. The specific heats of air, water vapour and liquid water at constant pressure can be taken as constants in the temperature range $0 < t < 100$°C, i.e. $C_{pl} = 1.00$ kJ kg^{-1} K^{-1}, $C_{pv} = 1.84$ kJ kg^{-1} K^{-1} and $C_{pl} = 4.18$ kJ kg^{-1} K^{-1}.

Hence, we can use $D_m(w, t^*)$ and $D_t(w, t^*)$ measured at a specified temperature t^* to determine the specific permeability $k(w)$ from equation (18), and to determine $B(w)$ from equation (19), then to calculate

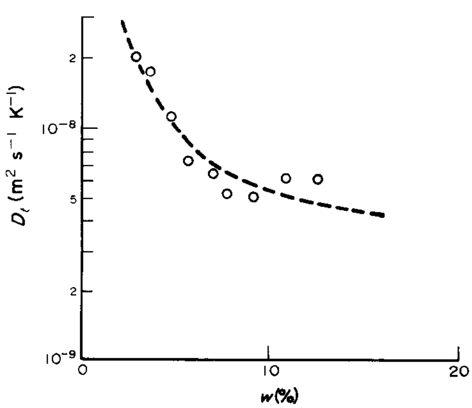

FIG. 2. The measured thermo-mass diffusivity of the sand at 20°C.

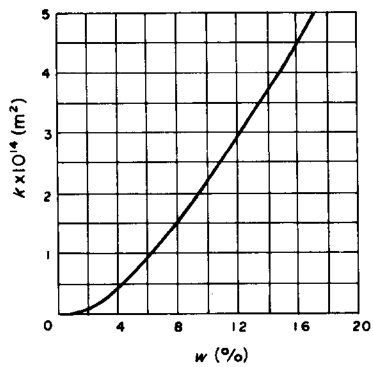

FIG. 3. The specific permeability of the wet sand.

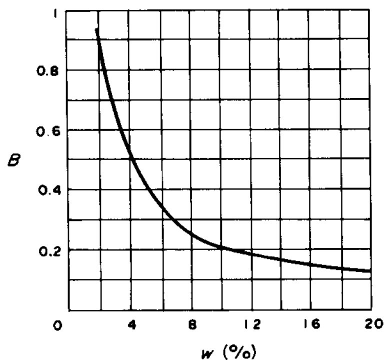

FIG. 4. The influence function of the wet sand.

$D_m(w, t)$, $D_t(w, t)$, $\xi(w, t)$ and $\alpha(w, t)$ according to equations (12)–(15).

It should be pointed out that, from equations (12) and (9), we have

$$\frac{D_m(w, t)}{D_m(w, t^*)} = \frac{\sigma(t)\mu(t^*)}{\sigma(t^*)\mu(t)}. \quad (23)$$

4. WORKING EXAMPLE

The mass diffusivity $D_m(w, t^*)$ and thermo-mass diffusivity $D_t(w, t^*)$ of the wet sand had been measured at $t^* = 20$°C, and summarized in Figs. 1 and 2 [7, 10]. The grain diameter of the sand sample tested was 0.25–0.5 mm, with porosity 0.41. The saturated moisture content was 0.273, and its matrix nominal density was 1510 kg m^{-3}.

Taking $p_0 = 3247$ N m^{-2}, $n = 0.137$ [9], and the data of $D_m(w, t^*)$ and $D_t(w, t^*)$ shown by the dashed lines in Figs. 1 and 2, we calculated $k(w)$ and $B(w)$ from equations (18) and (19), respectively, which had been plotted in Figs. 3 and 4, while the evaluated mass diffusivity $D_m(w, t)$, thermo-mass diffusivity $D_t(w, t)$, non-linear thermal conductivity $\xi(w, t)$ and mass–thermal conductance $\alpha(w, t)$, with temperature t ranging from 10 to 90°C, are shown in Figs. 5–8. It is found that, all these properties are sensitive to both temperature and moisture content, so that the effect of varying them should be considered in general.

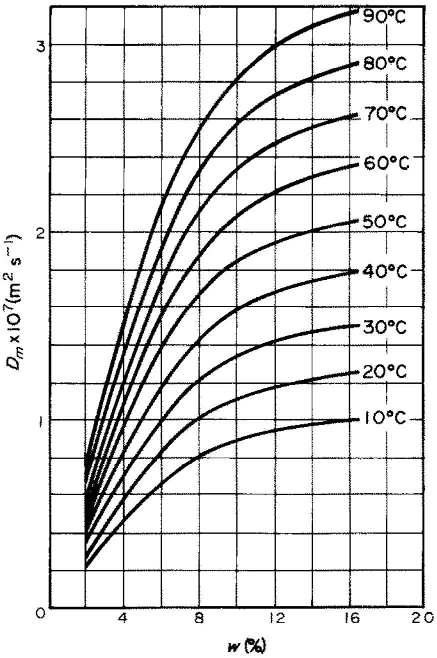

FIG. 5. The evaluated mass diffusivity of the wet sand.

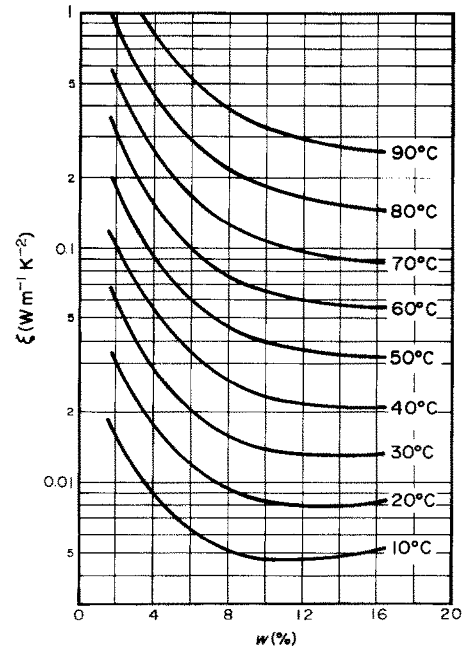

FIG. 7. The evaluated non-linear thermal conductivity of the wet sand.

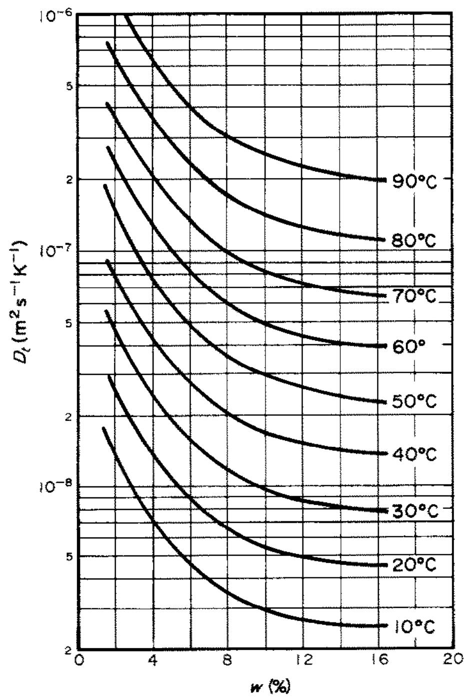

FIG. 6. The evaluated thermo-mass diffusivity of the wet sand.

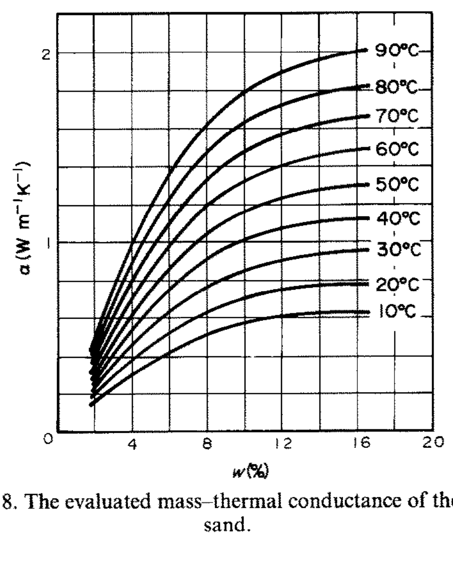

FIG. 8. The evaluated mass–thermal conductance of the wet sand.

5. CONCLUDING REMARKS

The semi-empirical method proposed here can be used to evaluate the heat and mass transport properties for meeting the needs in engineering practice. This may be an economic approach, which can save time and investments for tedious experiments. The reliability of this method depends mainly upon the accuracy of the data $D_m(w, t^*)$ and $D_t(w, t^*)$, and also depends upon the precision of the capillary pressure $p_c(w, t)$.

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REFERENCES

1. D. Hillel, *Introduction to Soil Physics*. Academic Press, New York (1982).
2. A. V. Luikov, *Theoretical Basis for Architectural Thermophysics* (in Russian). B.S.S.R. Academy of Science, Minsk, U.S.S.R. (1961).
3. A. V. Luikov, *Heat and Mass Transfer in Capillary-porous Media*. Pergamon Press, Oxford (1966).

4. B. X. Wang and Z. H. Fang, Heat and mass transfer in wet porous media and a method proposed for the determination of the moisture transport properties, *Heat Technol.* **2**(1), 17–34 (1984).
5. B. X. Wang and W. P. Yu, On the heat and mass transfer in moist porous media. In *Heat Transfer Science and Technology* (Edited by B. X. Wang), pp. 602–608. Hemisphere, Washington, DC (1987).
6. W. P. Yu, A synthetical analysis and experimental study of heat and mass transfer in moist porous media (in Chinese), M.S. Thesis, Tsinghua University, Beijing (1984).
7. B. X. Wang, L. Z. Han and W. P. Yu, A method for measuring simultaneously the heat and mass transport properties of moist porous media, *Int. J. Expl Heat Transfer, Thermodyn. Fluid Mech.* (1988), in press.
8. B. X. Wang and Z. H. Fang, Water absorption and measurement of the mass diffusivity in porous media, *Int. J. Heat Mass Transfer* **31**, 251–257 (1988).
9. J. Bear, *Dynamics of Fluids in Porous Media*. American Elsevier, New York (1972).
10. W. P. Yu, The heat and mass transport properties of moist porous media (in Chinese), Dissertation for Doctoral Degree, Tsinghua University, Beijing (1987).
11. M. W. Zemansky and R. H. Dittman, *Heat and Thermodynamics*, 6th Edn. McGraw-Hill, New York (1981).
12. E. R. G. Eckert and R. M. Drake, Jr., *Analysis of Heat and Mass Transfer*. McGraw-Hill, New York (1972).

METHODE D'EVALUATION DES PROPRIETES DE TRANSFERT DE CHALEUR ET DE MASSE D'UN MILIEU HUMIDE POREUX

Résumé—On propose un méthode d'évaluation de la diffusivité de masse $D_m(w, t)$, de la diffusivité de chaleur $D_t(w, t)$, de la conductivité thermique non linéaire $\xi(w, t)$ et de la conductance masse-chaleur $\alpha(w, t)$ pour des milieux poreux à différentes températures et à différentes humidités, ceci à partir de la mesure de $D_m(w, t^*)$ et de $D_t(w, t^*)$ faite à une température de référence t^*. Cette approche est tres appréciable pour la pratique industrielle.

EINE METHODE ZUR BESTIMMUNG DER WÄRME- UND STOFF-TRANSPORTEIGENSCHAFTEN VON FEUCHTEN PORÖSEN MEDIEN

Zusammenfassung—Es wird eine Methode zur Bestimmung verschiedener Wärme- und Stofftransporteigenschaften von feuchten porösen Medien bei unterschiedlichen Temperaturen und Feuchtegehalten vorgestellt. Das Verfahren beruht auf der Messung der Diffusionskoeffizienten $D_m(w, t^*)$ und $D_t(w, t^*)$ bei einer bestimmten Referenztemperatur t^*. Es ist eine brauchbare und bequeme Näherung für den praktischen Gebrauch.

МЕТОД ОЦЕНКИ ХАРАКТЕРИСТИК ТЕПЛО- И МАССОПЕРЕНОСА ВЛАЖНЫХ ПОРИСТЫХ СРЕД

Аннотация—Предложен метод оценки массопроводности $D_m(w, t)$, тепломассопроводности $D_t(w, t)$, нелинейной теплопроводности $\xi(w, t)$ и массотеплопроводности $\alpha(w, t)$ влажных пористых сред при различных температурах и влагосодержаниях, основанный на измерении массопроводности $D_m(w, t^*)$ итепломассопроводности $D_t(w, t^*)$ при заданной температуре t^*. Этот подход удобен для инженерной практики.